

Mid-Term Examination

Algebra II

M.Math.

1st March, 2010

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Instructions. All questions carry equal marks. \mathbb{Q} denotes the field of rational numbers.

1. Let F be a finite extension of \mathbb{Q} . Prove that the algebraic closure of F and \mathbb{Q} are isomorphic to each other.
2. Let K be a finite separable extension of F . Let G denote the group of automorphisms of K over F . Prove that $\{x \in K \mid g(x) = x, \forall g \in G\}$ equals F .
3. Describe the degrees of the splitting fields over \mathbb{Q} of the polynomials $X^3 + 2X + 1$ and $X^5 - 3$. Justify your answer.
4. Let f be a degree 6 irreducible polynomial over a field F . Let K be a degree 2 extension of F . Prove that either f is irreducible over K or is a product of two irreducible cubic polynomials over K .
5. Define separable degree of a field extension. Prove that the separable degree of a finite extension is bounded by the degree of that extension.
6. Prove that -1 is not a square in the field $\mathbb{Q}(\sqrt[4]{-2})$.